Math 2150 - Homework # 6
Second order linear ODEs - Theory

1. Use the Wronksian to show that the following functions are linearly independent on I.

   (a) $f_1(x) = x,\ f_2(x) = 3x^2,\ I = (-\infty, \infty)$

   (b) $f_1(x) = \sin(2x),\ f_2(x) = \sin(x),\ I = (-\infty, \infty)$

   (c) $f_1(x) = \frac{1}{x},\ f_2(x) = x^2,\ I = (0, \infty)$

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2. In this problem we will solve

$$x^2y'' - 5xy' + 8y = 24$$

   on the interval $I = (-\infty, \infty)$.

   (a) Show that $y_h = c_1x^2 + c_2x^4$ is the general solution to the homogeneous equation $x^2y'' - 5xy' + 8y = 0$.

   (b) Show that $y_p = 3$ is a particular solution to $x^2y'' - 5xy' + 8y = 24$.

   (c) Give a formula for the general solution to $x^2y'' - 5xy' + 8y = 24$.

   (d) Find the solution to the initial-value problem

$$x^2y'' - 5xy' + 8y = 24, \quad y'(1) = 0,\ y(1) = -1$$

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3. In this problem we will solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x} + 4x - 12$$

   on the interval $I = (-\infty, \infty)$.

   (a) Show that $y_h = c_1e^{2x} + c_2xe^{2x}$ is the general solution to the homogeneous equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

(b) Show that $y_p = x^2 e^{2x} + x - 2$ is a particular solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x} + 4x - 12$.

(c) Give a formula for the general solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x} + 4x - 12$.

(d) Find the solution to the initial-value problem

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x} + 4x - 12, \quad y'(0) = 0,\ y(0) = 1$$

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4. In this problem we will solve

$$2x^2y'' + 5xy' + y = x^2 - x$$

on the interval $I = (0, \infty)$.

(a) Show that $y_h = c_1x^{-1/2} + c_2x^{-1}$ is the general solution to the homogeneous equation $2x^2y'' + 5xy' + y = 0$.

(b) Show that $y_p = \frac{1}{15}x^2 - \frac{1}{6}x$ is a particular solution to $2x^2y'' + 5xy' + y = x^2 - x$.

(c) Give a formula for the general solution to $2x^2y'' + 5xy' + y = x^2 - x$.

(d) Find the solution to the initial-value problem

$$2x^2y'' + 5xy' + y = x^2 - x, \quad y'(1) = 0,\ y(1) = 0$$
